

Probing the pseudogap phase of cuprates by a Giaever transformer

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We develop a theory of the rectification effect in a double-layer system where both layers are superconductors, or one of the layers is a normal metal. The Coulomb interaction is assumed to provide the dominant coupling between the layers. We find that superconducting fluctuations strongly enhance the drag conductivity, with rectification most pronounced when both layers are superconductors. In view of their distinct dependence on temperature near T_c and layer separation, drag measurements based on a Giaever transformer could distinguish whether rectification occurs due to fluctuating pairs or inductively coupled fluctuating vortices.

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Introduction— The origin of the pseudogap phase in high temperature cuprate superconductors is the subject of on-going debate [1]. One possible explanation of this phase is the presence of preformed Cooper pairs which deplete the density of states near the Fermi level [2]. This is consistent with photoemission studies which reveal that the Fermi surface breaks up into disconnected arcs [3] that should have profound implications for the nature of transport in the pseudogap phase [4].

In this connection, recent observations [5, 6] of a large Nernst signal in the pseudogap phase has generated much interest. Two competing theories may account for this phenomenon. On the one hand, in proximity to the superconducting transition temperature T_c , it is natural to attribute a large Nernst response to fluctuating vortex-like excitations which carry entropy. On the other hand, there exists a close correspondence of the Nernst signal with the fluctuational diamagnetism [7], which points towards the Nernst effect as originating from fluctuating pairs. Existing theories based on Ginzburg-Landau or diagrammatic approaches [8–10] give a good description of the Nernst data. It is thus desirable to have an additional probe of the pseudogap phase which can distinguish transport contributions arising from fluctuating vortices or preformed pairs. In this paper, we suggest using a Giaever transformer [11] for such a probe and develop the theory for this.

Measurements on double-layer systems [12] have provided us invaluable information about the scattering mechanisms underlying transport. A typical experiment consists of driving one of the subsystems (active layer) and measuring an induced response in the other one (passive layer). Physically, by driving a current in the active layer, one creates an electromagnetic environment to which the passive layer is exposed. These fluctuations are rectified, thus producing a finite drag response. Under the condition of no tunneling between the layers, momentum can be transferred only via electron-electron scattering due to the inter-layer Coulomb interaction. In contrast, if vortices are present, then inductive coupling between the layers becomes important. Experimentally, the

measured quantity is the drag resistance, $R_D = V_D/I$, which is the ratio between the induced voltage V_D in the passive layer to the driving current I in the active one. The temperature dependence of $R_D(T)$ is extremely sensitive to the microscopic properties of the system. For example, in Fermi liquids $R_D \propto T^2$, which results from the phase space available for scattering. In contrast, in disordered systems $R_D \propto T^2 \ln T$ [13], while for non-Fermi liquids, like composite fermions in quantum Hall bi-layers, the drag exhibits a very different temperature dependence, namely $R_D \propto T^{4/3}$ [14]. Many other interesting examples can be found in the literature [15].

In recent work [16], the rectification effect was studied in the context of a magnetic field driven superconductor-insulator transition and attributed to fluctuating vortices. With some modifications, this theory can be applied to the case of cuprates under consideration here. However, we take the point of view that the dominant contribution to the rectification signal in the pseudogap phase is due to fluctuating pairs. Below, we develop the corresponding theory within a linear response Kubo formalism.

Formalism— We consider both a symmetric superconductor bi-layer and a non-symmetric one where one layer is replaced by a normal metal. The building blocks of the theory are the normal and superconductor Greens functions

$$G_n(p, i\varepsilon) = \frac{1}{i\bar{\varepsilon}_n - \xi_p}, \quad G_s(p, i\varepsilon) = -\frac{i\bar{\varepsilon}_s + \xi_p}{\bar{\varepsilon}_s^2 + \xi_p^2 + \Delta_p^2}. \quad (1)$$

In what follows we assume a free electron dispersion relation, $\xi_p = p^2/2m^* - \varepsilon_F$, which in two dimensions implies a constant density of states, simplifying the energy integrations. We assume a momentum dependent d-wave pseudogap of the form $\Delta_p = \frac{\Delta}{2}[\cos(p_x) - \cos(p_y)] \simeq \Delta \cos 2\vartheta_p$. In Eq. (1) $\bar{\varepsilon}_{n(s)} = \varepsilon + \gamma_{n(s)} \text{sign}(\varepsilon)$, with $\gamma_s \simeq T$ being the scattering rate observed by photoemission in cuprates, and $\gamma_n = 1/2\tau_n$ a constant scattering rate assumed for the normal metal.

For the fluctuating Cooper pairs in a superconductor, we take the standard expression for the propagator whose

retarded component has the form [17]

$$L^R(Q, \Omega) = -\frac{1}{N_s} \frac{1}{\pi D_s Q^2 / 8T + \epsilon - i\pi\Omega / 8T + \Upsilon_\Omega} \quad (2)$$

where $\Upsilon_\Omega = \frac{\Omega}{2\Delta} \frac{\partial \Delta}{\partial \epsilon_F}$ accounts for electron-hole asymmetry, N_s is the density of states, D_s the diffusion constant and $\epsilon = \ln(T/T_c)$.

We assume a screened Coulomb potential as the leading source of coupling between the layers

$$U(q, \omega) = \frac{\pi e^2 q}{\kappa_{n(s)} \kappa_s \sinh(qd)}. \quad (3)$$

Here we have neglected retardation effects. d is the spacing between the layers and $\kappa_{n(s)} = 4\pi e^2 N_{n(s)}$ is the Thomas-Fermi screening momentum in the normal metal (superconductor). Applying a linear response analysis [13] to a Giaever device (Fig. 1a), we have for the drag conductivity

$$\sigma_D = \frac{1}{16\pi T} \sum_q \int_{-\infty}^{+\infty} \frac{d\omega |U(q, \omega)|^2}{\sinh^2 \frac{\omega}{2T}} \bar{\Gamma}_{n(s)}(q, \omega) \bar{\Gamma}_s(q, \omega) \quad (4)$$

where $\bar{\Gamma}_{n(s)}$ is the nonlinear susceptibility (rectification coefficient) within each layer. Knowing σ_D and inverting the conductivity tensor, one finds the drag resistivity $\rho_D = \sigma_D / (\sigma_n \sigma_s)$, and finally the drag resistance $R_D = \rho_D (\ell/w)$, with ℓ and w being the length and width of the two-dimensional strip. In the normal metal, the rectification coefficient is given by (Fig. 1b)

$$\begin{aligned} \bar{\Gamma}_n(q, i\omega_1, i\omega_2) &= eT \sum_{p, \varepsilon_l, \pm} \vec{v}_p G_n(p_\pm, i\varepsilon_l) \\ G_n(p_\mp, i\varepsilon_l \pm i\omega_1) G_n(p_\mp, i\varepsilon_l \pm i\omega_2) \end{aligned} \quad (5)$$

with $p_\pm = p \pm q/2$, and $\varepsilon_l = 2\pi T(l + 1/2)$ being the fermionic Matsubara frequency. The susceptibility $\bar{\Gamma}_n(q, \omega)$ which enters Eq. (4) is obtained from Eq. (5) upon appropriate analytic continuation $i\omega_{1,2} \rightarrow \omega \pm i0$ such that $\bar{\Gamma}_n(q, \omega) \equiv \bar{\Gamma}_n^{RA}(q, \omega, \omega)$, where indices $R(A)$ stand for the retarded (advanced) components in the complex frequency plane. In the superconducting layer we have (Fig. 1c)

$$\begin{aligned} \bar{\Gamma}_s(q, i\omega_1, i\omega_2) &= 4eT \sum_{Q, \Omega_m, \pm} \vec{B}(Q_\pm, i\omega_{1,2}, i\Omega_m) \\ A(Q, q, i\Omega_m + i\omega_1, i\Omega_m) A(Q, q, i\Omega_m, i\Omega_m + i\omega_2) \\ L(Q_\mp, i\Omega_m) L(Q_\pm, i\Omega_m \pm i\omega_1) L(Q_\pm, i\Omega_m \pm i\omega_2) \end{aligned} \quad (6)$$

with $Q_\pm = Q \pm q/2$ and $\Omega_m = 2\pi mT$ being the bosonic Matsubara frequency. The scalar (Coulomb) vertex function defining $\bar{\Gamma}_s$ has the explicit structure

$$\begin{aligned} A(Q, q, i\Omega_m + i\omega_1, i\Omega_m) &= T \sum_{p, \varepsilon_l} G_s(p, i\varepsilon_l) \\ G_s(Q - p, i\Omega_m + i\omega_1 - i\varepsilon_l) G_s(p - q, i\varepsilon_l - i\omega_1) \end{aligned} \quad (7)$$

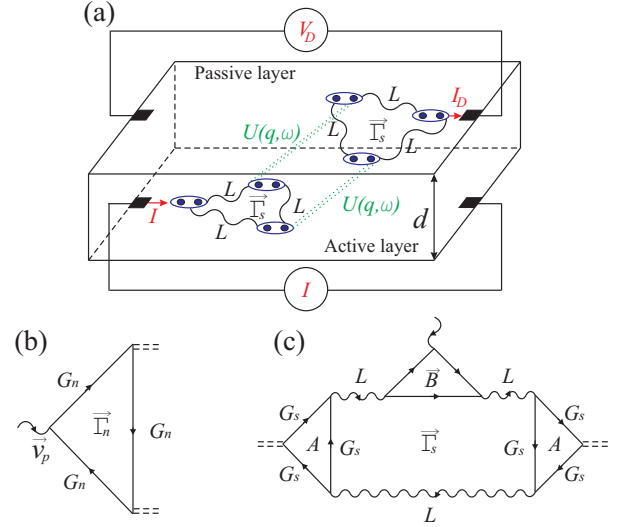


FIG. 1: (Color online) (a) Schematic representation of the rectification effect due to preformed Cooper pairs in a double-layer superconductor Giaever device. Diagrams (b) and (c) depict the rectification coefficients $\bar{\Gamma}_{n(s)}$ in the normal metal and the superconductor, respectively.

while the current vertex reads

$$\begin{aligned} \vec{B}(Q, i\omega_{1,2}, i\Omega_m) &= T \sum_{p, \varepsilon_l} \vec{v}_p G_s(p, i\varepsilon_l) \\ G_s(p, i\varepsilon_l - i\omega_1 + i\omega_2) G_s(Q - p, i\Omega_m + i\omega_1 - i\varepsilon_l) \end{aligned} \quad (8)$$

Similar to the previous case, $\bar{\Gamma}_s(q, \omega)$ entering Eq. (4) is obtained from Eq. (6) after analytic continuation to the real energy axis $i\omega_{1,2} \rightarrow \omega \pm i0$. From Eq. (6) one sees an analogy between rectification and the Nernst effect. Indeed, the Nernst is given by a Feynman graph of three currents connected by fluctuation propagators similar to Fig. 1c. The rectification coefficient $\bar{\Gamma}_s$ has the same structure with the only difference being that two of the current vertices are replaced by a scalar field which couples to the electromagnetic potential in the other layer. This point is illustrated in Fig. 1a, which schematically represents the rectification effect due to fluctuating pairs. In order to find σ_D , we need an explicit form for $\bar{\Gamma}_{n(s)}(q, \omega)$ which we derive below.

Results– We start from Eq. (5) and convert the Matsubara sum over the fermionic frequency into the contour integral $T \sum_{\varepsilon_l} \rightarrow \oint_C \frac{d\varepsilon}{4\pi i} \tanh \frac{\varepsilon}{2T} G_n(\varepsilon) G_n(\varepsilon \pm i\omega_1) G_n(\varepsilon \pm i\omega_2)$, where the momentum variables of the Greens functions have been suppressed for brevity. The contour is a circle with three branch cuts at $\text{Im}(\varepsilon) = \{0, -\omega_1, -\omega_2\}$. The outer branches of the contour that have all three Greens functions of the same causality do not contribute. The remaining parts, after analytic continuation, give us

in the ballistic case

$$\vec{\Gamma}_n(q, \omega) = \frac{e\omega}{\pi} \sum_{p, \pm} \vec{v}_p \text{Im}[G_n^R(p \mp q, \varepsilon_F)] |G_n^R(p, \varepsilon_F \pm \omega)|^2 \quad (9)$$

where we assumed $\int_{-\infty}^{+\infty} d\varepsilon [\tanh \frac{\varepsilon + \omega}{2T} - \tanh \frac{\varepsilon}{2T}] = 2\omega$ (low temperature limit). We next expand the Greens function to linear order in \vec{q} and perform the remaining momentum integration by approximating $\sum_p \vec{v}_p (\vec{v}_p \cdot \vec{q}) \dots = N_n \vec{q} \int_{-\infty}^{+\infty} d\xi_p \frac{\varepsilon_F + \xi_p}{m^*} \dots$ which accounts for the curvature of the electronic dispersion. Since $\max\{\omega, q^2/2m^*\} \ll \varepsilon_F$ we find to leading order

$$\vec{\Gamma}_n(q, \omega) = 4e \frac{D_n \vec{q} N_n \omega}{\varepsilon_F v_F q} \quad (10)$$

which agrees with earlier calculations [13]. One important remark here is that for the linearized dispersion, $\xi_p = v_F p$, rectification vanishes due to electron-hole symmetry.

We proceed with the derivation of $\vec{\Gamma}_s(q, \omega)$, taking into account a temperature dependent scattering rate $\gamma_s \simeq T$ appropriate for cuprates [Eq. (1)]. Starting from Eq. (6) we convert the bosonic Matsubara sum into the contour integral $T \sum_{\Omega_m} \rightarrow \frac{1}{4\pi i} \oint_C d\Omega \coth \frac{\Omega}{2T} A(\Omega + i\omega_1, \Omega) A(\Omega, \Omega + i\omega_2) L(\Omega) L(\Omega \pm i\omega_1) L(\Omega \pm i\omega_2)$ with three branch cuts at $\text{Im}(\Omega) = \{0, \mp\omega_1, \mp\omega_2\}$. After analytic continuation and expansion of the fluctuational propagator [Eq. (2)] to leading non-vanishing order in the electron-hole asymmetry Υ_Ω , we find

$$\vec{\Gamma}_s(q, \omega) = \frac{4eN_s}{\pi} |A|^2 \vec{B}(q) \sum_Q \int_{-\infty}^{+\infty} d\Omega \Upsilon_\Omega \mathcal{F}(\Omega, \omega) \quad (11)$$

$$|L^R(Q, \Omega)|^2 \text{Re}[L^R(Q, \Omega)] \text{Im}[L^R(Q, \Omega + \omega)] \quad (11)$$

where $\mathcal{F}(\Omega, \omega) = \coth \frac{\Omega + \omega}{2T} - \coth \frac{\Omega}{2T}$. When deriving this expression, we made use of the following key observations. (i) Near T_c the characteristic bosonic energies are smaller than the fermionic ones $\{DQ^2, \Omega, \omega\} \sim T - T_c \ll T$. This allows us to compute the vertex functions [Eqs. (7)-(8)] by neglecting their dependence on the bosonic modes. This step is legitimate since both A and \vec{B} contain a fermionic loop with $\varepsilon \sim T$. (ii) Under the above approximation, one then finds from Eqs. (7) and (8) with the help of Eq. (1)

$$|A|^2 = \frac{N_s^2}{\Delta^2} \alpha^2(\gamma_s/\Delta), \quad \vec{B}(q) = -\frac{\pi N_s D_s \vec{q}}{4\Delta} \beta(\gamma_s/\Delta) \quad (12)$$

where scaling functions

$$\alpha(z) = \frac{1}{16\pi} \left[\frac{1}{\sqrt{1+z^2}} + 3 \ln \left(\frac{1 + \sqrt{1+z^2}}{|z|} \right) \right] \quad (13)$$

$$\beta(z) = \frac{\pi^2 \xi^2}{24\eta} \left[\frac{z^2 + 2}{z\sqrt{z^2 + 1}} - 1 \right] \quad (14)$$

account for the suppression of the vertex due to the pseudogap [10]. In Eq. (14), $\xi = \hbar v_F / \pi \Delta$ and $\eta = \pi D / 8 \Delta$.

(iii) We also exploited the fact that $Q \gg q$ and set $q \rightarrow 0$ in the propagators since rectification is determined by small momentum transfer between the layers, namely $Dq^2 \sim (T - T_c) / (\kappa_{n(s)} d) \ll T - T_c \sim DQ^2$. (iv) Our final comment regarding Eq. (11) concerns the electron-hole asymmetry. In general Υ_Ω enters both the fluctuational propagator L and the scalar vertex A which, in fact, is a consequence of gauge invariance [13, 18]. We find that the most singular in $T - T_c$ contribution to σ_D comes from keeping Υ_Ω in L while taking the scalar vertex as the bare one. The technical reasoning is that by expanding L in Υ_Ω , one effectively raises the power of the fluctuational propagator that in the end translates into an extra factor of ϵ in σ_D after energy integrations. Conversely, keeping Υ_Ω in A would also lead to a nonvanishing contribution to $\vec{\Gamma}_s$, but with a subleading dependence on $T - T_c$ near the transition.

We can now simplify the integrand in Eq. (11) even further by exploiting the separation of the energy scales $\{\Omega, \omega\} \ll T$, thus approximating $\mathcal{F}(\Omega, \omega) \approx -\frac{2T\omega}{\Omega(\Omega + \omega)}$ and using Eq. (2) which gives us

$$\vec{\Gamma}_s(q, \omega) = -\frac{32eT^2}{\pi^2 N_s^3} \vec{B}(q) |A|^2 \frac{\partial \ln \Delta}{\partial \varepsilon_F} \sum_Q \int_{-\infty}^{+\infty} d\Xi_\Omega \frac{\Lambda_Q \Xi_\omega}{[\Lambda_Q^2 + \Xi_\Omega^2]^2 [\Lambda_Q^2 + (\Xi_\Omega + \Xi_\omega)^2]} \quad (15)$$

Here we introduced dimensionless variables $\Xi_\Omega = \pi\Omega/8T$ and $\Lambda_Q = \pi D_s Q^2 / 8T + \epsilon$. The remaining energy and momentum integrations can be completed analytically with the final result

$$\vec{\Gamma}_s(q, \omega) = -\frac{4eT^2}{\pi^2 D_s N_s^3 \epsilon^3} \vec{B}(q) |A|^2 \frac{\partial \ln \Delta}{\partial \varepsilon_F} \frac{\omega}{4 + (\omega \tau_{GL})^2} \quad (16)$$

where the Ginzburg-Landau time $\tau_{GL} = \pi / (8T\epsilon)$ was introduced. By combining Eqs. (10) and (16) in Eq. (4), we find the drag conductivity due to fluctuational rectification by preformed Cooper pairs in the non-symmetric normal metal-superconductor transformer (restoring \hbar)

$$\frac{\sigma_D}{\sigma_Q} = \frac{\pi^2 r_s^2}{30} \frac{\partial \Delta}{\partial \varepsilon_F} \frac{\lambda_{ns}(d) f(T)}{\ln^2 \frac{T}{T_c}} \quad (17)$$

where $\sigma_Q = \frac{e^2}{\hbar}$, $\lambda_{ns}(d) = \frac{(D_n / v_F d)}{(\kappa_n d)^2 (\kappa_s d)^2}$, $f(T) = \left(\frac{T}{\Delta}\right)^4 \alpha^2 \beta$ and $r_s = \frac{e^2}{\hbar v_F}$. In the immediate vicinity of the transition, $T - T_c \ll T_c$, where $f(T)$ is constant, the rectification has a strong power-law enhancement, $\sigma_D \propto \left(\frac{T_c}{T - T_c}\right)^2$, which physically corresponds to an Aslamazov-Larkin (AL) paraconductivity effect [19]. Notice that σ_D is more singular than the AL conductivity $\sigma_{AL} \propto \frac{T_c}{T - T_c}$. This situation is similar to the Hall effect in superconductors [17, 18] since both the drag and Hall conductivities are sensitive to the electron-hole asymmetry and require an expansion of L in Υ_Ω , while the AL conductivity is not.

Further away from the transition, the vertex suppression due to the pseudogap, where $\alpha \propto \Delta/T$, $\beta \propto (\Delta/T)^2$ and $f(T) \sim 1$, results in a moderate decay of $\sigma_D \propto \frac{1}{\ln^2 \frac{T}{T_c}}$. Screening effects on the Coulomb potential provide an additional suppression of σ_D for large separation between the layers.

In the case when both layers are superconductors, we use two vertices from Eq. (16) in Eq. (4) and find

$$\frac{\sigma_D}{\sigma_Q} = \frac{15\zeta(5)r_s^2}{32} \frac{\partial \Delta_1}{\partial \varepsilon_F} \frac{\partial \Delta_2}{\partial \varepsilon_F} \frac{\lambda_{ss}(d)g(T)}{\ln^2 \frac{T}{T_{c1}} \ln^2 \frac{T}{T_{c2}} \left[\ln \frac{T}{T_{c1}} + \ln \frac{T}{T_{c2}} \right]} \quad (18)$$

where $\lambda_{ss}(d) = \frac{(\xi_1 \xi_2 / d^2)}{(\kappa_{s1} d)^2 (\kappa_{s2} d)^2}$, $g(T) = \left(\frac{T^2}{\Delta_1 \Delta_2} \right)^3 \alpha_1^2 \alpha_2^2 \beta_1 \beta_2$ with shorthand notation $\{\alpha, \beta\}_{1,2} = \{\alpha, \beta\}(\gamma_s / \Delta_{1,2})$. This result shows a similar but potentially more singular behavior near the transition as Eq. (17), however, with a much faster power-law decay at higher temperatures, $\sigma_D \propto \frac{1}{T^2 \ln^5 T}$, since $g(T) \propto (\Delta_1 \Delta_2 / T^2)$ for $T > \max\{T_{c1}, T_{c2}\}$. This high- T suppression mechanism due to the pseudogap was recently proposed as an explanation of the fast fall-off of the Nernst signal in underdoped cuprates [10].

Summary— In conclusion, we developed a theory of a Giaever transformer based on bi-layers of cuprate superconductors, assuming that the leading source of rectification originates from fluctuating Cooper pairs. The coupling between the layers is provided by a screened Coulomb interaction. The drag signal shows a strong singularity near the superconducting transition due to the paraconductivity effect, while the pseudogap leads to a faster drop-off at higher temperatures. Such a characteristic temperature dependence may distinguish drag due to preformed pairs from that due to vortices. In particular, Ref. 20 predicted that the drag resistivity ρ_D in a normal-superconductor bilayer due to the inductive coupling effect of vortices should follow the flux flow conductivity σ_v of the superconductor, $\rho_D \propto (\hbar/e^2)^2 \sigma_v$. Clearly, this result is very different from the contribution to ρ_D originating from fluctuating Cooper pairs [Eq. (17)]. The general condition to have a vortex drag effect is to have an inhomogeneous magnetic field in the barrier region (otherwise, the vortices between the two layers cannot ‘find’ each other). This means that the vortices must be well defined. As a consequence, any vortex drag regime above T_c should have a narrow temperature range. This is especially true for cuprates, given the weak coupling of vortices between layers (i.e., pancake vortices). That coupling gets even weaker for underdoped compounds, which is exactly the regime we are interested in.

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